Mean-Variance Optimal Stopping for Geometric Lévy Processes

Kamille Sofie Tågholt Gad
University of Copenhagen
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Given a geometric Levy process, X, I study the optimal stopping problem

$$\sup_{\tau} \left(\mathbb{E}[X_\tau] - c\mathbb{V}[X_\tau] \right),$$

where $c > 0$ and the supremum is taken over all stopping times generated from X. This problem I denote *the mean-variance problem*.

The idea of maximizing expectation while minimizing variance is familiar in economic and financial applications and dates back to Markovich, however not in the optimal stopping setting.

The mean-variance problem is a non-linear optimal stopping problem, and thus it falls outside the scope of classical optimal stopping problems. I first present how this problem for spectrally negative geometric Levy processes may be solved by first partially solving an auxiliary classical optimal stopping problem. Afterwards, I discuss shortly why the mean-variance problem is a lot more difficult to solve when the process, X, has upwards jumps. I solve it for a specific Cramer-Lundberg process with upwards jumps, and the solution is found to be the first entry to an interval.

From the proof of the mean variance problem follows the solution to the problem

$$\inf_{\tau:\ \mathbb{E}[X_\tau] \geq M} \mathbb{V}[X_\tau].$$

It is remarkable that for some starting values we find that this problem of minimizing variance is solved by a randomized stopping time.